# Algebra I
## Mid Term Examination

**Instructions:** All questions do not have same level of difficulty. However, they all carry equal (non-zero) marks. Justify all your answers.

**1.** Explicitely define an equivalence relation on the real plane $\mathbb{R}^2$ whose equivalence classes are lines parallel to the line $y = x$.

**2.** Let $m$ and $n$ be natural numbers with $d$ being their largest common divisor. Prove that any other common divisor of $m$ and $n$ must divide $d$.

**3.** Let $(G, \bullet)$ be a group. Define a new binary operation $\circledcirc$ on $G$ by $g \circledcirc h = h \bullet g$. Show that $(G, \circledcirc)$ is a group and prove that it is isomorphic to $(G, \bullet)$.

**4.** Let $C_n$ denote the cyclic group of order $n$. Prove that for every divisor $d$ of $n$, there exists a unique subgroup of $C_n$ of order $d$ and find the number of elements of order 8 in the group $\mathbb{Z}/88888\mathbb{Z}$.

**5.** Let $G$ be a group containing a unique element of order 2, say $g_0$. Prove that $g_0$ commutes with every element of $G$.

**6.** Prove that every group of order 6 must have an elelment of order 2 and an element of order 3.

**7.** Define a cycle of length $l$ in $\mathcal{S}_n$, the group of permutation on $n$ letters. Prove that order of such a cycle is $l$ and that every element of $\mathcal{S}_n$ is a product of disjoint cycles.

**8.** Classify all groups of order $1, 2, 3, 4$, and 5.